Searching for Patterns and Similarities in Road Traffic with the Use of Modern Techniques

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Abstract

The article presents the possibilities of using new methods to determine patterns and similarities in road traffic. Modern technological development enables the processing of a large number of data in a given space and short time. Thus, it becomes possible to identify phenomena which until now could not be recognized and described in a reasonable way. The disclosed self-similarities in the road network can support effective organization and control of traffic.

Keywords: Regulated Traffic, Clusters, Patterns, Big Data.

1. Introduction

In the broadly understood transport, traffic processes can be divided basically into two categories of movement: regulated and self-regulating. Regulated traffic occurs in railway transport, where successive rail vehicles are launched in accordance with the adopted traffic timetable and occupy strictly defined sections of the railway network. For safety reasons, in the vast majority of cases, one rail vehicle occupies one element of the railway network. Certain "deregulation" of this movement arises as a result of human errors, failures, unfavorable weather conditions and concerns a relatively large number of rail vehicles. For example, for the Polish railway network, average delays are currently above 5 minutes (Office of Rail Transport data). Nevertheless, a certain irregularity of this movement is taken into account (calculated) in the traffic timetable, taking into account the appropriate time for a fixed length of the railway line (train running).

In contrast to railway traffic, road traffic is a self-regulating movement [1], where, apart from public transport and in a certain range of goods, there is no strictly fixed timetable of movement of vehicles. In the case of passenger cars, such a timetable, even if it exists in obligatory motivations (time of arrival and time of leaving) or is routed using planning tools [2], is not determined by the accuracy known from typical timetables for public transport (1 or 0.5 minute for the public transport timetable, ± 0.1 minutes for the technical timetable in the rail service). In such a movement, fluctuations in the time of generation and absorption of travel can be strong already on a weekly basis (counting day to day). The technical infrastructure of transport in road traffic is not isolated / dedicated to a single vehicle, which results in the formation of interactions, not only in nodes but also on road sections. This causes that the "original" lack of regularity in the area of road traffic, associated with the lack of a strict timetable, multiplies depending on the structure of traffic and its parameters. Traffic control systems that organize traffic in spatially limited areas of their presence are a limitation in this area. As a result of the operation of traffic control systems (traffic lights), the arrangement of traffic flows through their packetisation

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(grouping, creation of so-called platoons) occurs discreetly in the space of the road network [3], [4]. After leaving the area of interaction of traffic lights, there is a further increase in the irregularity of this traffic in the form of so-called dispersion of traffic flow. The farther from the intersection are the vehicles it is greater.

The phenomena related to the occurrence of road events and changing weather conditions occur in the road traffic accumulation processes. Although the temporary effect of a traffic incident is an increase in the homogeneity of traffic (vehicles waiting to unblock the traffic in the queue behind the event site are characterized by a similar distance to homogeneous), due to delays in traffic, in these cases we have to increase irregularities on subsequent sections of the network traffic (compensation of time losses). The increase in traffic irregularity is also related to the fact that vehicles change traffic relations in the road network bypassing traffic congestion resulting from a road accident. In this way, they increase the irregularity of traffic in other streams (unusual traffic in this place of the network).

The regularity of traffic occurs when a vehicle appears in a certain place of the road (cross-network) at a specific time, according to the expected timetable or planned travel timetable (1):

$$\mathrm{MR} = \Delta \mathrm{l_S} \Delta \mathrm{t_S} \qquad (1)$$

MR	-	Measure of traffic regularity	[m*s]
Δl_S	-	Difference in the distance between the scheduled / planned position of the vehicle in the road network in section S, and the actual location. It can be measured directly, accurately using the WGS 84 system	[m]
Δt_S	-	The time difference between the scheduled / planned arrival of the vehicle to the S section of the road network and the estimated time of arrival. It can also be measured using satellite receivers (accuracy up to 1 second)	[s]

Thus determined (1) (multiplicative model) the regularity of traffic can be determined at any point of the road network. This equation can be extended by much more factors for example expectation arrival time to final destination. Such a record takes into account and emphasizes the heterogeneity of the description of the road network in terms of time and space. Due to the fact that travel time in the road network is not road equivalent (displacement time depends on the parameters of traffic flows, distances are immutable), which results from the presence of variable traffic conditions, the equation (1) uses the product (multiplicative model).

In the classical approach (which is a simplification of dependence (1)), a certain approximation of irregularities is a measure of traffic congestion, which can be defined as a waste of time expressed in difference [5]:

$$\Delta = \mathrm{t_S} - \mathrm{t_m} \qquad (2)$$

Δ	-	Waste of time	[s]
t_S	-	Average driving time of a vehicle with a specified road section (actually measured)	[s]
t_m	-	Minimum driving time on the same section of road with free flow speed, in the absence of other vehicles that may interfere with traffic, no stops resulting from a change in the status of traffic lights, etc.	[s]

Equation (2) indicates how much time the vehicle loses on the road network, due to the occurrence of circumstances accompanying its displacement. This is also a measure of traffic irregularity. These values are related, for example, to the value of the expected time of arrival at the travel destination, which is related to the behavioral factors of perceiving current traffic conditions.

The smaller the MR (1) value, the greater the regularity of traffic in the S section (in this case it is the average value for all vehicles that will cross this section in a specific direction of movement):

$$MR_S = \frac{\sum_i^n \Delta l_S \Delta t_S}{n} \tag{3}$$

MR_S	-	The average measure of traffic regularity in the cross-section	[m*s]
Δl_S	-	The difference in the distance between the scheduled / planned position of the vehicle in the road network in the S section and the actual location can be measured directly, accurately using the WGS 84 system	[m]
Δt_S	-	The time difference between the scheduled / planned arrival of the vehicle to the S section of the road network and the estimated time of arrival	[s]
n	-	The number of vehicles observed in the S cross-section of the network	[-]

Irregularity of traffic defined in the area of the road network, congestion rate, as well as other spatial distributions, i.e. describing temporary locations of vehicles, places where road accidents occur, are a measure of chaos (being the opposite of regularity / ordering of traffic) that occurs in the road network. In deterministic models describing real world systems (eg traffic models), the minimal change in conditions usually leads to proportional changes in the output data from the model [6]. In the real world, the traffic image can be diametrically different from the model predictions due to the appearance of a small change in parameters, such as the collision of two vehicles, road closure, signal failure, especially in an important traffic flow, behavioral expectations of driver.

Therefore, road traffic, apart from the situations of road congestion, is essentially heterogeneous and strongly random. However, the question arises whether there are certain similarities in this heterogeneity? This article points to several potential techniques for searching for similarities in road traffic.

Road traffic is implemented (carried out) in a certain strictly defined structure of the road network, which is most often formally represented in the form of a directed graph. In the case of a directed graph, there is a mathematical apparatus (matrix algebra) allowing its formal description. With respect to the description of the traffic structure (streams and traffic flows), you can also use matrix algebra. The structure of the road network is constant in medium and short time horizons. Road traffic described in the form of flows and streams of movement is not characterized by this property of immutability in short intervals of time. Thus, there are two structures in the description of the road network, one static in long periods of time and the other dynamic. A dynamic description of the traffic structure requires a description often characterized by significant data redundancy. The bigger, the greater the registered heterogeneity of road traffic. Taking into account the dynamics of traffic in the medium-sized city road network in a 24-hour period means the need to describe traffic with Big Data datasets.

In this context, a legitimate question should be asked, is there another, simpler way of describing the structure of road traffic? Is there a description of the traffic that does not require the use of multi-dimensional traffic tables for all the intervals and traffic directions in which this traffic is not homogeneous? Can this description also refer to the variability of the generic structure of the traffic, whose variance in small ranges is important for the traffic image and possible algorithms of its control? Could this be related to the description of the traffic structure, etc.? Therefore, a different way of describing of road traffic and its dynamics in the road network for its static structure is sought.

This article is an attempt to answer the question of whether the road traffic structure can be subjected to such compression and / or extrapolation, which uses small sets of data in the process of describing traffic parameters. Perhaps there are simplified ways to describe traffic in the road network, especially globally and holistically. It boils down to looking for existing patterns in the road network in terms of observed traffic volumes, the distribution of vehicle queues, time losses, locations of road accidents, etc. To what extent are motion structures subject to machine learning, a description of fractal geometry, or any spatial patterns in them? and time? This article is a kind of introduction to the description of a wide range of related problems and a certain attempt to answer these questions carefully.

2. Patterns and Structures of Traffic

Road traffic observed from the air perspective (drones, UAVs, airplanes, satellite system and others) allows for obtaining a certain spatial image (planar graphic representation), where it can be seen that individual vehicles create different types of patterns, clusters and clusters.

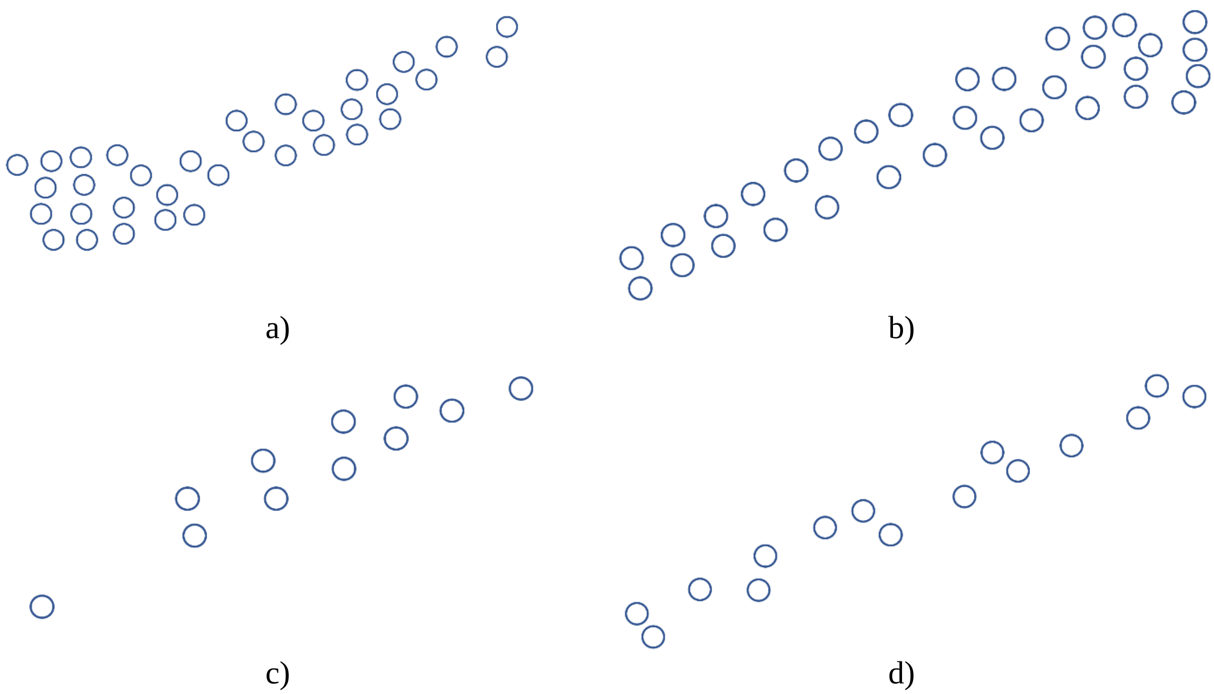


Figure 1 Clusters observed in the traffic flow of vehicles in road traffic. Source: own research based on photos from the air.

In Figures 1a (inlet E) and 1b (inlet W) an image of road traffic is presented on two different inlets of the same traffic node (these are opposite inlets with similar geometry and the same traffic organization). The vehicle position mapping presented in Figures 1a and 1b was made for one moment of time. One can see in these figures (1a and 1b) a certain difference in the traffic image on both opposite inlets (number of vehicles: 34 and 33). There are also visible clusters of points representing the location of individual vehicles. The spacing between the wheels represented on these two drawings representing individual vehicles results not only from the traffic structure (different generic structure of traffic) but also from the different way of moving around the lane of subsequent vehicles (other spacing between individual vehicles). Thus, such a way of analyzing a traffic flow carries more information than a classic traffic intensity test. In this picture, the intervals between vehicles and the generic structure of the traffic are also recorded, also some information about the behavioral nature (comparing different moments of observation of these two inlets). Fig. 1c-d presents the traffic image on two opposite inlets of one traffic node (other than in Figures 1a and 1b).

In this case, you can also see a characteristic distribution of clusters of vehicles and the occurrence of some differences between the two inlets (vehicle ratio: 11 to 15). The more traffic there is on the observed inlet, the greater the differences in vehicle distribution are observed. In analyzing such a traffic image, pattern recognition can be used. This type of algorithms conducts the classification of measurement data to identify characteristic patterns based on information obtained statistically. Such images as presented in Figs 1a-d can be further subjected to statistical analysis. Patterns define the position of the corresponding points in space (depending on the observed feature). In order to implement such an approach to recognition of traffic parameters, the following steps should be taken:

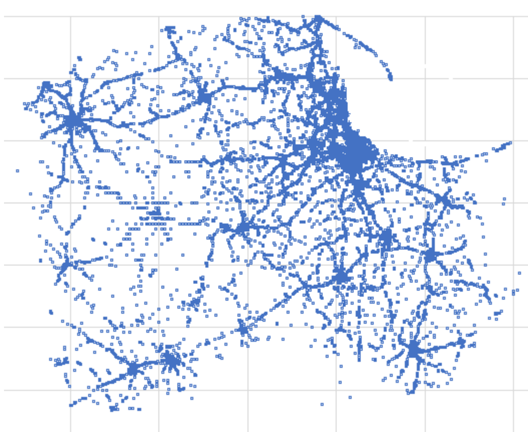
- Determination of a measuring apparatus that provides observations subject to further classification (video recording),
- Using one of the mechanisms for extracting features that best describe and separate from the environment a given class of objects,
- Mapping the information extracted from the image into a symbolic figure,
- Use of a symbolic form, i.e. to control the parameters of traffic lights.

Although patterns occur in almost all images, in this case the repeatability of these patterns is important in time and space, also between different nodes of the network. At the same time, such information, as noted earlier, carries greater information capacity than classical measurements of vehicle traffic intensity, queue length measurement, measurement of driving times, etc. According to the authors, this type of video traffic recording functionality should be used in traffic control systems, because it allows to take into account the greater dynamics of variation of traffic parameters than other such methods. This approach better defines each traffic flow separately from individual parameter values.

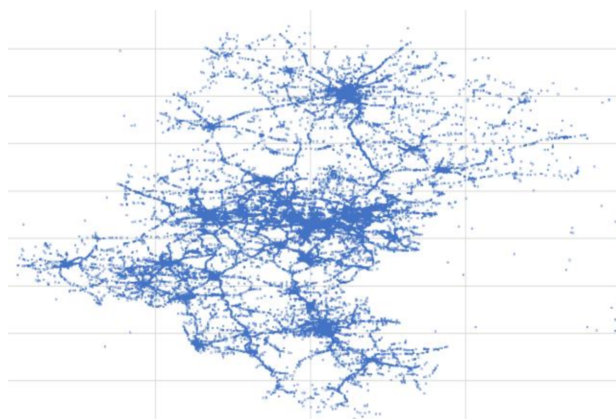
The above presented traffic images (albeit static, captured at a given moment) can be used in a natural way, after machine processing, to control the traffic of vehicles at intersections equipped with traffic lights.

In relation to the road network, there are also data sources that do not have the same dynamics as the presented motion images (those shown in Fig. 1 a-d can be changed with a frequency of up to several hundred fps). Such data include, for example, data describing events in the road network. These data describe the road network in a static view with the date of the occurrence of a specific event in road traffic (date and time, place of occurrence). Nevertheless, this data can also be used in the process of organizing road traffic. The longer the analysis period of this type of data, the more reliable its character is, and the homogeneous distribution.

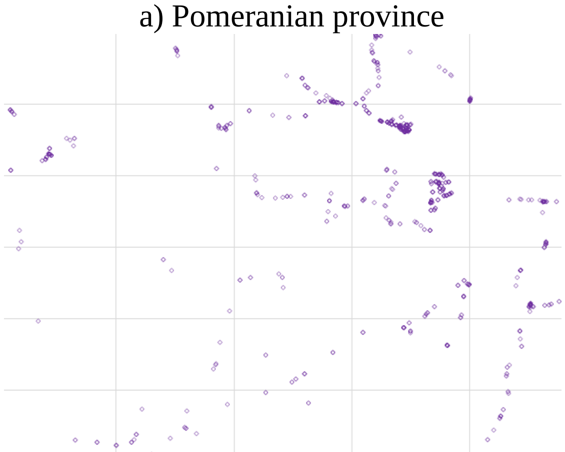
Fig. 2a and 2b show the distribution of traffic events in two provinces in Poland: Pomeranian and Silesian. Figures 2c and 2d present the same data limited to the event category only: traffic accidents occurring at specified distances from the railroad crossings. In each of these four cases (Fig. 2 a-d), a characteristic data distribution is closely correlated with the spatial structure of the road network. An image presenting the full distribution of all traffic incidents (Fig. 2a and b) may not be readable, as the characteristic points cover almost exactly all sections of the analyzed road networks.



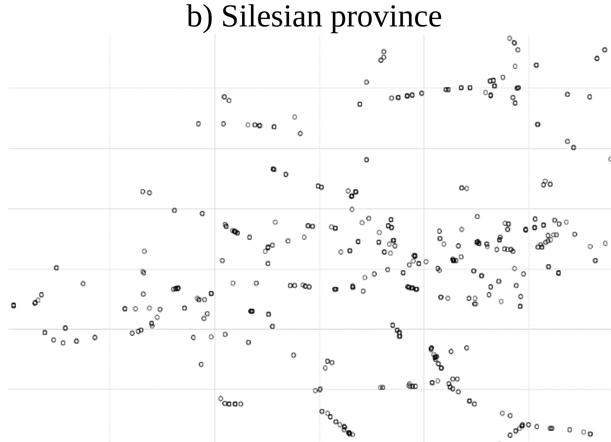
a) Pomeranian province



b) Silesian province



c) Pomeranian province



d) Silesian province, level crossing-related events.

Figure 2 Visualization of road events in Poland in 2016 using GIS technology. Source: own research based on datsets from SEWIK (The System of Registering of Accidents and Collisions; in Polish) 2016.

Even if patterns are observed, they are poorly visually identifiable. Narrowing the data from Figures 2a and b to the data presented in Fig. 2c and d (choosing the event category from the set of all events with the addition of an additional criterion, i.e. location of the occurrence place near the railroad crossing) allows to eliminate this drawback of the method.

In this case, the characteristic patterns in these drawings are revealed. Interestingly, clusters of certain points occur on such an image away from transport nodes and can be an impulse for changes in traffic organization. In contrast to the recognition of images of traffic flows presented in Fig. 1a-d, in this case there is a relatively static picture of the traffic status and it also requires a series of computationally complex transformations to extract the necessary information. Nevertheless, as the authors point out, such analyzes are suitable for introducing changes in the organization of road traffic (in wide transport systems area, in opposite to node control systems).

Another way of analyzing traffic images is to look for fractal structures (searching for fractal structures can be found in various areas of life [7], [8], [9]). Even a cursory analysis of Figures 1a-d and 2c and 2d shows some similarities in the traffic structures (air traffic images) and the distribution of points related to the occurrence of traffic events. There is also a certain repeatability of their occurrence in the road network space. To what extent this repeatability is determined by the structure of the network, and to what extent the specificity of the observed characteristics requires further research using the description of fractal structures. Some similarities in road traffic images and their repetition in the space of the road network are presented in Figures 3a-d (the similarities were marked with appropriate colors or contour).

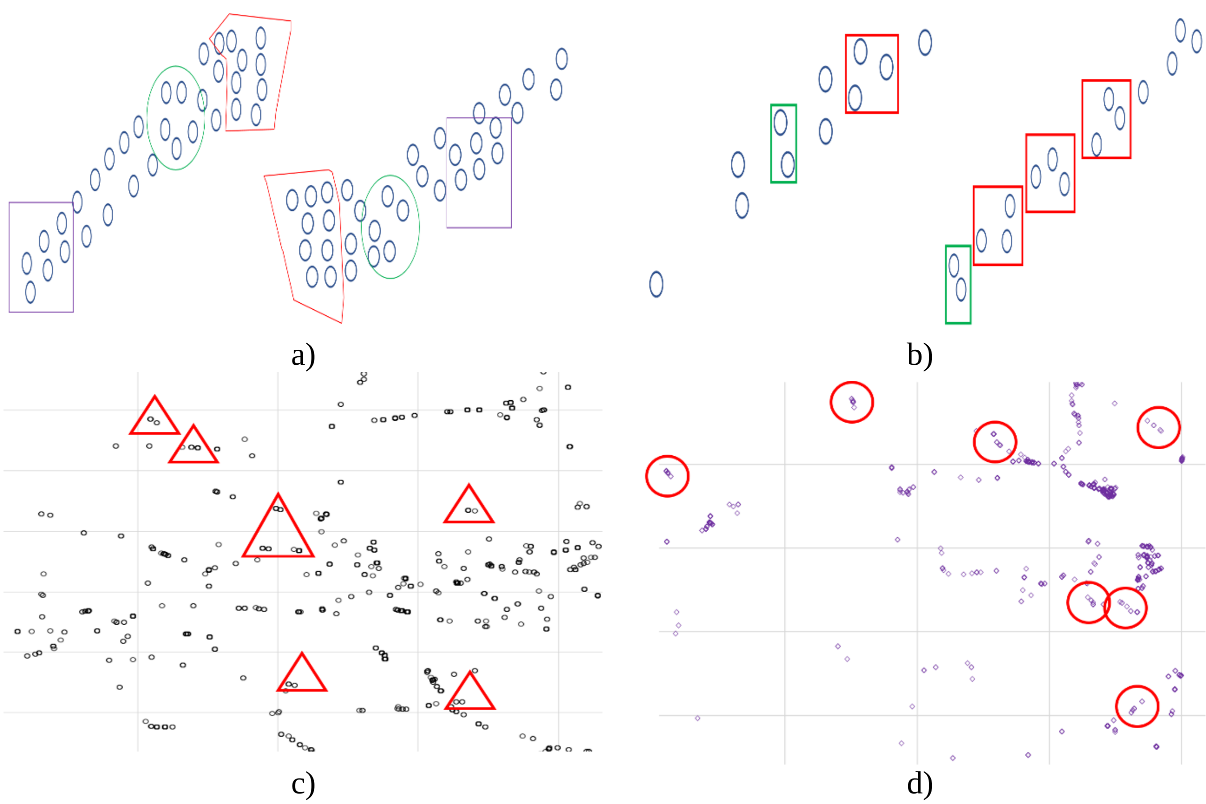


Figure 3 Visual similarities in data structures related to traffic event space distributions. Source: own research.

Fig. 3a-b shows that certain characteristic positions of vehicles in the traffic flow of vehicles on different inlets of the same node are repeated. These are very similar clusters of mutual positioning of more vehicles than one. Fig. 3c-d shows that some characteristic systems of location of road events in the vicinity of railroad crossings are repeated in the larger space of the road network (a characteristic arrangement of points, suggesting their repetition in space and time, with different distances between them). What's more, similar systems of connected places of occurrence of road events are visible for completely different elements of transport infrastructure (different road geometry and number of lanes etc.). The question arises: why, an event in traffic, as a random event, causes the occurrence of uniform distances between them, or distances characterized by a certain pattern? Why do such events seem to create certain sequences in the range of their mutual distance? These and other questions require a future computational effort to explain the nature of these phenomena. This is justified by the existence of patterns in the absence of similarity in the structure of the road network.

The initial and very general answer to this question can be answered - and why should it not be so? In nature, fractal forms are observed both in relation to objects of animate nature and inanimate nature. In the first case, especially tree and bush structures are characterized by a certain degree of self-similarity [10]. It is worth noting that the structure of the road network (itself) resembles tree forms, where each node understood as a junction has specific branches equipped with leaves ("inlets and outlets as leaves") etc. These specific leaves are also vehicles found in moment of time at particular inlets of the intersection. There is, therefore, only a certain similarity to the fractals, because such structures, unlike them, are limited, although this also depends on the scale of the area, which will be taken into account in the analysis of the road network. A priori, it can be assumed that in the case of the road network, the scaling of its area, rather sooner than later, causes the loss of the self-similarity. There is a significant difference observed here, especially in relation to mathematical phracts (unnatural, abstract). Such a fractal is characterized by a self-similar structure during the process of reducing their size going to the infinity [11]. In the case of the road network, the extent of such a fractal (if any) will depend heavily on: node structure, time of day, and traffic flow characteristics in the node.

The question arises whether the similarity of the structure of the road network to natural ones is similar to the structure of traffic. I mean, are traffic structures in the road network not self-similar? Positive verification of such a hypothesis could result from the method / form of the description of the road traffic structure enabling the compression of traffic data in large areas of the road network. Immediately raises another question. If there is such a method of describing road traffic in the network, to what extent is it accurate and how large areas of the road network apply? Can it be used to control traffic lights that organize traffic (by default, changes observed patterns in the road network)? According to the authors, for this purpose you can use the methods of processing mobile images, which was shown in the first part of the article (Figures 1a-d). Perhaps the effect of signaling, manifesting itself in the form of a dispersion of traffic flows, also creates self-similar structures in the road network, but of a different type (rolling up and expand the structure of traffic)? Is it possible that self-similar structures exist within the limits of a single traffic flow? Can it exist in individual generic groups of vehicles? Such questions can be multiplied and this is the main purpose of this review article, nevertheless finding answers to them can allow for a significant simplification of procedures regarding the organization and control of traffic in road networks.

Fractals can generally be divided into mathematics (Koch's curve, Sierpinski's carpet, Cantor's collection) and natural ones (eg sea coast lines, trees, bushs, etc.) [12], [13], [14]. In the structure of natural fractals, there are only statistical similarities, they refer to the proportions of elements and their number, not exactly the same arrangement of elements in each scale of observation of a natural fractal. In contrast to natural fractals, mathematics are similar to each other (close self-similar) in each scale of observation. The scale of observation means the geometric scaling of the fractal image up (decrease) and down (enlargement of its fragment). More precisely, it can be said that natural fractals differ from the deterministic (mathematical) occurrence of only statistical self-similarity. Scaling up natural fractals causes the loss of ownership of their self-similarity. This property, and hence the similarity in terms of certain proportions, is interesting from the point of view of the problem of describing the road network undertaken in this article. The stochastic nature of road traffic precludes the exact self-similarity of large traffic structures. There are attempts to find similarities in the number of vehicles appearing in the road network, in their spatial distribution, in the presence of certain characteristic clusters depending on the scale of observation (the adopted area of analysis).

In the case of road traffic, it is important to find a scale or area for the road network in which the original fractal (description of the traffic structure) ceases to be a fractal. In addition to this scale of observation or outside the area of the road network, it loses the ownership of self-similarity. This is a significant research problem in the study of fractal traffic structure. In road traffic, such divisions of the road network into areas with similarity and without it are likely to occur. However, if these divisions do not significantly diversify the road network - they can become a legitimate tool for its (possibly better) organization and / or can be used in traffic control algorithms.

The question arises, what is the fractal dimension for the traffic structure in the road network? In the case of mathematical fractals, this dimension is calculated as the logarithm of the number of elements after division to the proportion of the basic dimension of the fractal. In road traffic, this should be adequately expressed by the proportion of the number or density of vehicles for different scale of road network division (on different sections). However, this procedure should be reversed in the road network. This means that the number N from equation (4) should be taken for up scaling of the observed object, not downward (ie increasing the observation scale).

$$D = \frac{\log N(h)}{\log(\frac{1}{h})} \tag{4}$$

where:

N- number of vehicles in the smallest scale of observation (scaling the structure of traffic to larger areas of the road network),

h- scale of observation.

3. Searching for Self-Similarities in Road Traffic

The authors' observations discussed earlier are strongly dependent on the spatial structure of the road network (more widely transport) and ways of its delimitation. Assume for the purpose of illustration one lane at each inlet of the intersection. For such assumptions, an example of a fragment of the road network will look as shown in Figures 4, 5 and 6.

In Fig. 4, the similarity of the number of vehicles occurring at the intersections of an isolated intersection is examined. The similarity to the number of these vehicles at the intersections of intersections surrounding them (in total). At the same time, it is possible to assume centricity as well as asymmetrical axial scaling for the study. The aim of the study is to find self-similar systems in the road network, in a different scale of its observation (for different scaling methods).

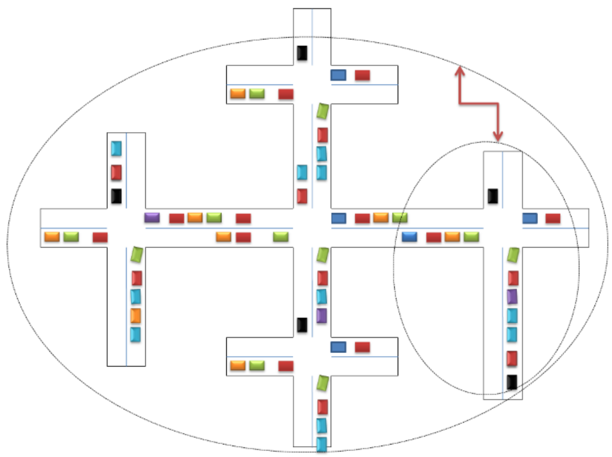


Figure 4 Traffic on the road network as a potentially scalable natural fractal: N1:N2. Source: own research.

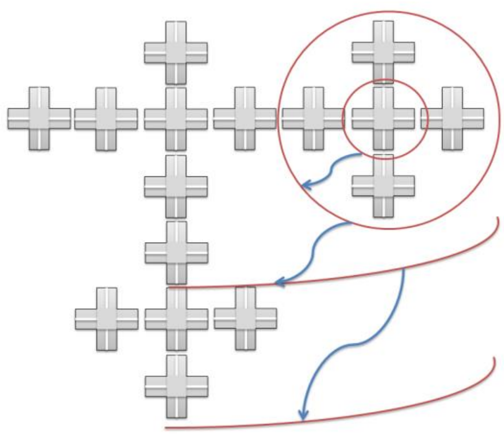


Figure 5 Traffic on the road network, as a potential scalable fractal nature, scale proportional area.

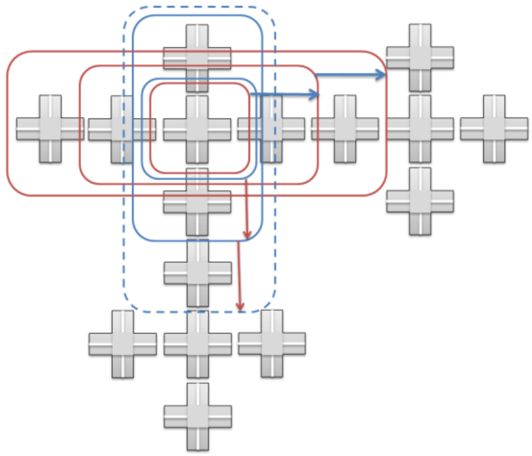


Figure 6 Traffic on the road network, as a potential natural fractal scalable, proportional scaling, linear (directional). Source: own research.

Figure 5 shows three-level centric scaling. In the first step, the self-similarity is determined for a single intersection (red circle). In the second step, the self-similarity is determined for the arrangement of five intersections, in relation to the intersection. In the next step, the procedure for determining self-similarity is reproduced. In contrast to those presented in Figures 4-6 of the theoretical arrangements of intersections, in real world, the distance between intersections and the number of lanes will also affect the self-similarity.

Figure 6 presents the study of self-similarities in road traffic in the so-called linear scale. In many networks, neighboring streets are run perpendicular or parallel. In Figures 5 and 6, examples of network scaling and sequential testing of self-similarities can be called area-specific. In this case, the scaling occurs in the direction of movement (in the direction of the main artery). It may occur by the expansion of the area or its enlargement towards the propagation of the main axis of motion.

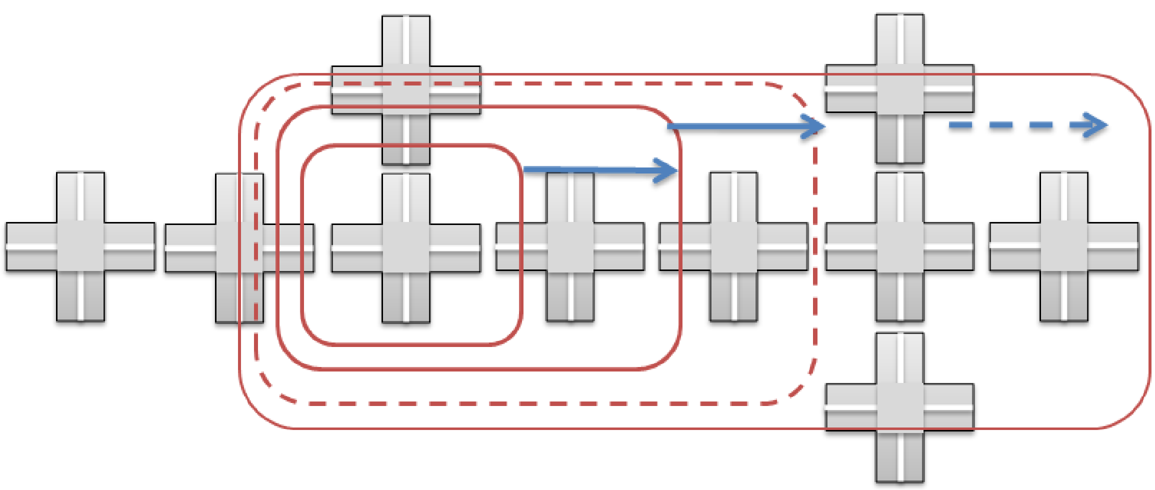


Figure 7 Traffic on the network, as a potential natural fractal scalable, proportional scaling, linear in the direction of the main artery.

Examination of single intersections does not require searching for self-similarities, because in this case, you can use the classical traffic test related to the measurement of intensity. However, already for tree structures of adjacent intersections and control systems in the area or area control, this type of research may provide additional information that may affect the optimization of the operation of traffic control algorithms.

4. Conclusions and Further Research

The idea of the article was to show an interesting direction for the development of the road network research methodology (more widely transport) in order to find patterns and self-similar structures in it. It is possible with the use of modern technologies and the involvement of appropriate computing powers that are able to process large data sets (Big Data). According to the authors, this type of knowledge will allow in the future to develop methods of organization and traffic control other than those used so far. In particular, the analysis of moving images (taken in the perspective of aerial photographs) is relatively easy to obtain data that can be used to control traffic. The fact that some similarities in the traffic image in the network are indicated by means of the illustrations in the article. They can also be found in selected publications, i.e. [15], [16], [17], [18]. The question as to the extent to which these structures occur in the entire area of the road network, what their spatial extent and what information capacity is. The article only indicates that this type of test method should also look for similarities not only in the number and distribution of vehicles, but also in terms of such characteristics as the generic and directional structure of traffic. It should be emphasized that in the case of a traffic image this does not refer to the analysis of traffic flow density, but the extraction of certain information and additional structures from this image.

Another question requiring research is whether there are similarities in railway traffic. It is a movement with essentially different characteristics. However, the detection of this type of similarity may allow the introduction of other methods of optimization in railway traffic than before. Methods based on spatial characteristics, which is important in the case of dense railway networks (i.e. Silesian conurbation).

Perhaps, such analyzes will allow to determine new methods for shaping a reasonable modal split in the road network [19]. It is a matter of identifying self-similar structures or clusters relating to specific types of transport. If such similarities are identified in the scope of the same types of vehicles, it will allow such an approach to optimize the modal split by showing atypical parts of the road network in this area.

Another interesting issue is the impact of traffic control on possible similarities in traffic structures over the network area. This can be used as a feedback (return channel) in traffic control algorithms [20]. Similarities in traffic structure can be used for adjustment traffic control systems.

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